



The Exergy Cost Theory

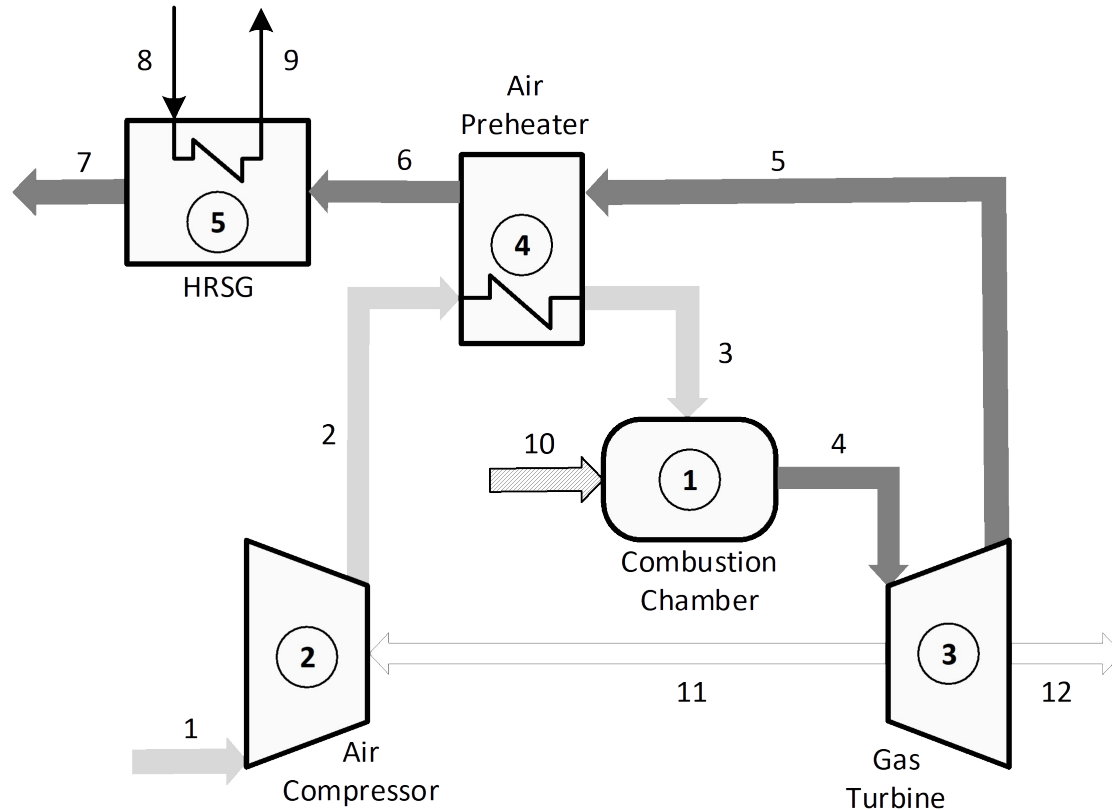
From Physical Irreversibility to Economic Accounting

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Objectives: From Physics to Economics

- Any plant or a chain of companies may be characterized by a graph
- Definition of Exergy Cost
- Define the concept of irreversibility
- Define exergy efficiency
- Overview of applications in exergetic engineering

A Plant as a Network: The CGAM Problem



- This graph is a set of components connecting by exergy flows
- The CGAM problem has been used since 1993, with the article: *"The CGAM problem. Definition and conventional solution"*
- It is a gas turbine cycle with an air preheater and a heat recovery steam generator to recycle the hot gases.

CGAM: Thermodynamic Properties

#	p (bar)	T(°C)	h (kJ/kg)	s(kJ/kg K)	m (kg/s)	B (kW)
1	1.013	25	298.4	6.86	104.8	0
2	8.611	315.4	595.4	6.939	104.8	28651
3	8.18	640.4	949.1	7.431	104.8	50339
4	7.771	1200	1605	8.004	106.2	102530
5	1.122	711.7	1029	8.087	106.2	38810
6	1.066	395.8	680.6	7.675	106.2	14784
7	1.013	147	421.8	7.207	106.2	2122
8	20	25	106.6	0.3664	10.21	19.43
9	20	212.4	2799	6.34	10.21	9322
10					1.449	72465
11						31105
12						30000

The F-P-R Framework

Notebook LM

$$F = P + I + R$$

F (Fuel): Exergy inputs required to drive a process (thermal, chemical, mechanical).

Production
Component

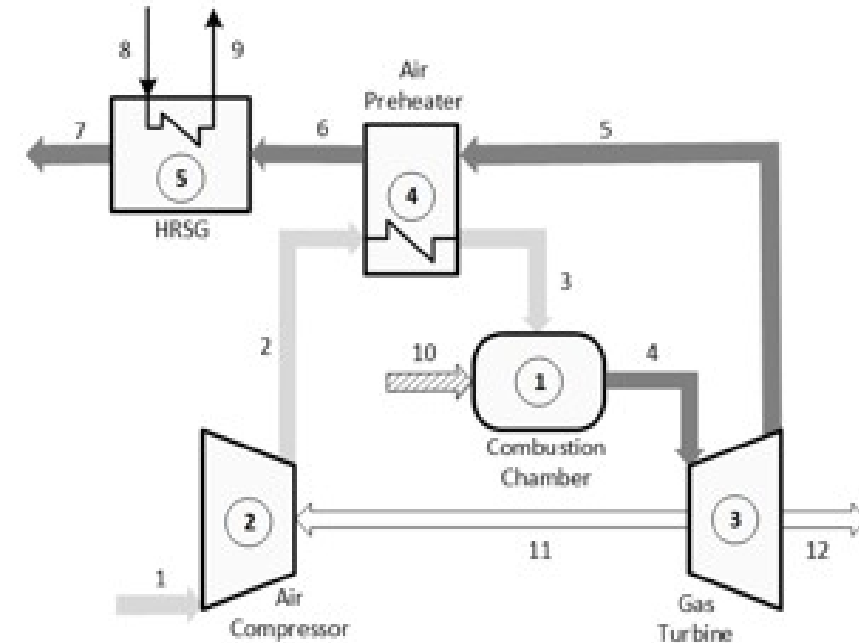
P (Product): The useful outputs generated.

R (Residue): Non-productive outputs (waste heat, emissions) transferred to the environment.

I (Irreversibility): Exergy permanently destroyed within the process.

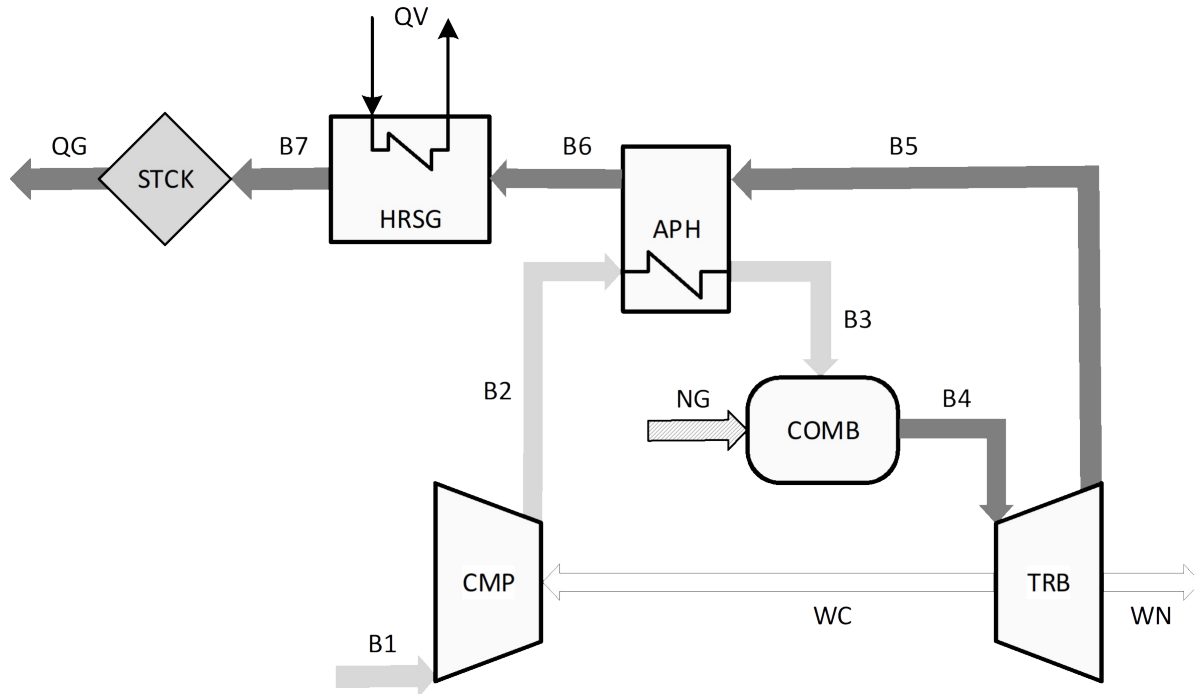
The Exergy Cost Theory: Production

Nr	Process	Fuel	Product	η (%)	k
1	Combustor	$B_{10} + B_3$	B_4	83.49	1.1977
2	Compressor	B_{11}	$B_2 - B_1$	92.11	1.0857
3	Turbine	$B_4 - B_5$	$B_{11} + B_{12}$	95.90	1.0428
4	Air Preheater	$B_5 - B_6$	$B_3 - B_2$	90.27	1.1078
5	HRSG	$B_6 - B_7$	$B_9 - B_8$	73.47	1.3611



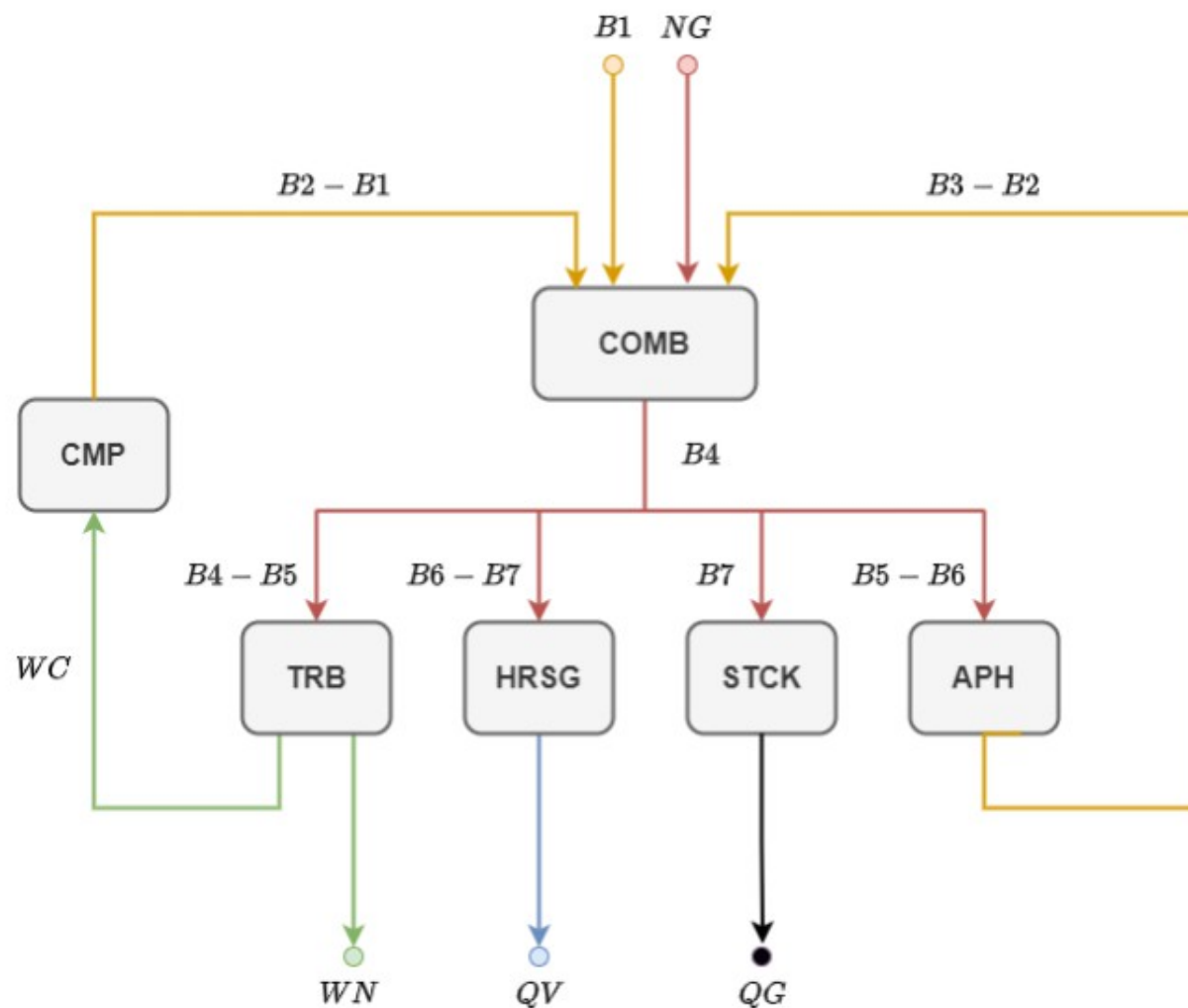
A product carries the physical burden of the resources consumed in its production.

CGAM Productive Structure



Nr	Process	Fuel	Product	Type
1	COMB	GN + B3	B4	PRODUCTIVE
2	CMP	WC	B2 – B1	PRODUCTIVE
3	TRB	B4 – B5	WC + WN	PRODUCTIVE
4	APH	B5 – B6	B3 – B2	PRODUCTIVE
5	HRSG	B6 – B7	QV	PRODUCTIVE
6	STCK	B7	QG	DISIPATIVE

Productive Diagram: CGAM System

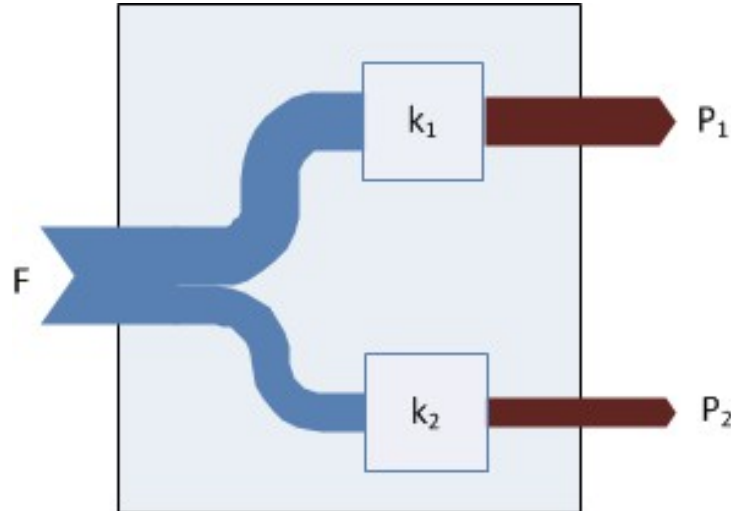


What Is Exergy Cost?

- **Given an energy system whose limits, level of aggregation, and productive structure have been previously defined. The exergy cost of each flow of the material or energy is defined as the amount of resources, measured in terms of exergy, necessary to obtain it.**
- **The exergy cost satisfies the following properties:**
 - **The cost of external resources must be known.**
 - **The cost of the waste produced by the system must be allocated to the processes that generated it.**
 - **The exergy cost is a conservative property. For each process, the cost of the resources used plus the cost of the waste generated must be assigned to the costs of its products and distributed proportionally to its exergy.**

Cost Allocation for Multiple Outputs

- If a process has several outflows, we need to determine the unit consumption of resources for each outflow.



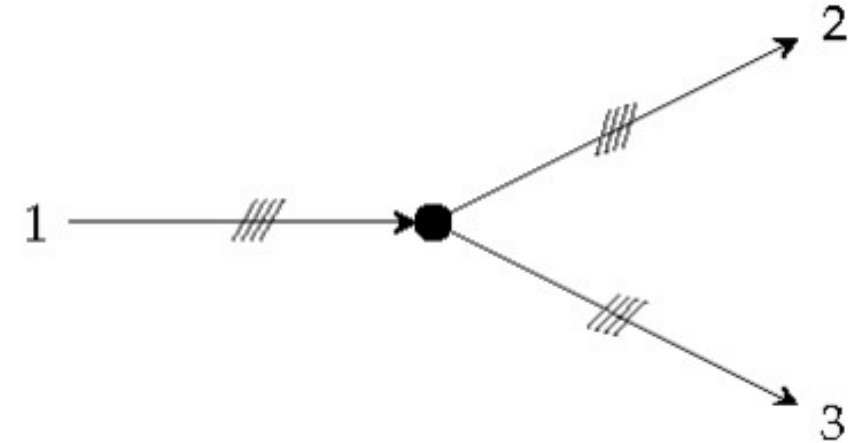
$$F = k_1 P_1 + k_2 P_2$$

$$k_1^* = k_F^* k_1$$

$$k_2^* = k_F^* k_2$$

The Co-Product Rule

- If two products are obtained simultaneously in a process, their costs are proportional to their exergy, and their unit costs are equal.



$$k = \frac{B_1}{B_2 + B_3}$$

$$y_2 = \frac{B_2}{B_2 + B_3}$$

$$y_3 = \frac{B_3}{B_2 + B_3}$$

The Recoverable Fuel Rule

- The output flows that are not part of the main product, but are considered as part of the recovered resources, have the same unit cost as the input resources.

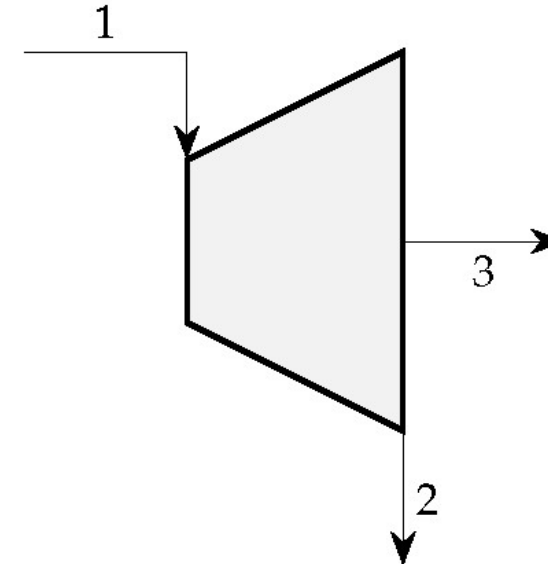
$$B_1 = B_2 + k B_3$$

$$k_2^* = k_1^*$$

$$k_3^* = k k_1^*$$

$$B_2^* = y_2 B_1^* = k_1^* B_2$$

$$B_3^* = (1 - y_2) B_1^* = k_1^* (B_3 + I)$$



$$k = \frac{B_1 - B_2}{B_3}$$

$$y_2 = \frac{B_2}{B_1}$$

Cost Allocation Rules

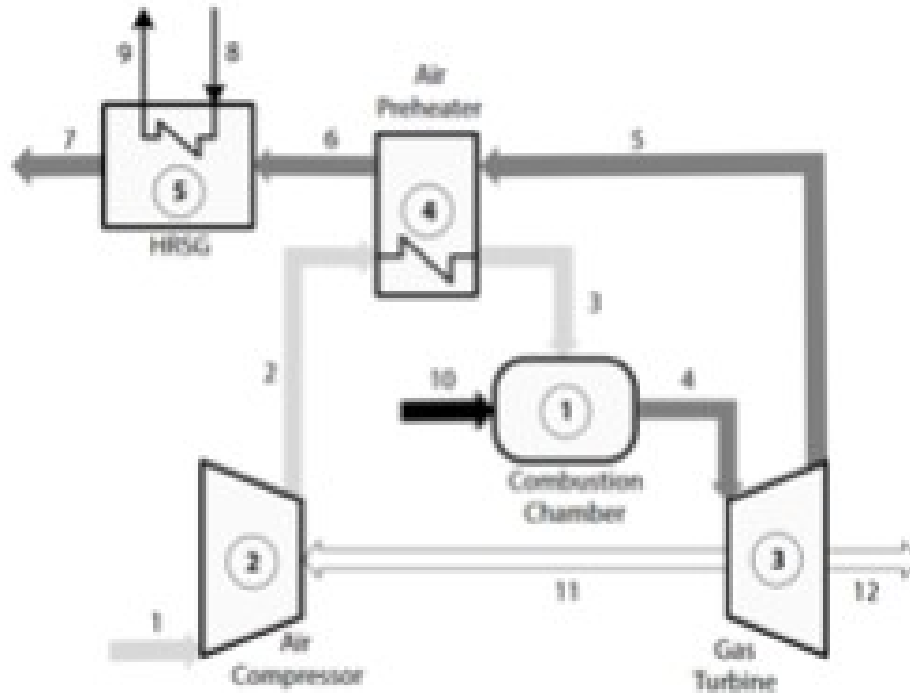
- **Resource Rule:** The exergy cost is relative to the system boundaries. In the absence of external valuation, the exergy costs of the resources is equal to their exergy.

$$B_i^* = B_i \quad i \in S_0$$

- **Cost Conservation Rule:** The exergy cost is a conservative property. This rule implies that the cost associated with a process, both the cost of resources expended, and the cost of waste generated, as well as the external costs associated with the processes, must be allocated to the cost of production.

$$P_u^* = F_u^* + R_u^* \quad u \in V$$

CGAM: Thermoeconomic Cost Equations



External Resources

$$B_1^* = B_1$$

$$B_8^* = B_8$$

$$B_{10}^* = B_{10}$$

Cost Balances

$$B_{10}^* + B_3^* - B_4^* = 0$$

$$B_1^* + B_{11}^* - B_2^* = 0$$

$$B_4^* - B_5^* - B_{11}^* - B_{12}^* = 0$$

$$B_2^* + B_5^* - B_3^* - B_6^* = 0$$

$$B_6^* + B_8^* - B_7^* - B_9^* = 0$$

$A \times B^* = 0$,
where Exergy costs
 B^* (GJ), of the Exergy
flows

Calculating Exergy Costs

- The cost balance matrix (n x m) is:

$$\mathbf{A} \equiv \mathbf{A}_P - \mathbf{A}_F - \mathbf{A}_R$$

- The resources matrix is:

$$\mathbf{a}_e \text{ (e x m)}$$

- The additional equations matrix (F-P rules) is:

$$\mathbf{a}_x \text{ (m-n-e x m)}$$

- The direct exergy cost is the solution of the system of equations:

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{a}_e \\ \mathbf{a}_x \end{pmatrix} \times \mathbf{B}^* = \begin{pmatrix} 0 \\ \mathbf{B}_e \\ 0 \end{pmatrix}$$

- The generalized exergy cost is the solution of the system of equations:

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{a}_e \\ \mathbf{a}_x \end{pmatrix} \times \mathbf{C} = \begin{pmatrix} \mathbf{Z} \\ \mathbf{v}_e \\ 0 \end{pmatrix}$$

CGAM: Matrix Calculation

$$\begin{bmatrix}
 0 & 0 & -1 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\
 -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
 0 & -1 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\
 \hline
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 \hline
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.964 & -1 \\
 0 & 0 & 0 & 0.378 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0.381 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0.144 & -1 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix} \times \begin{bmatrix} B_1^* \\ B_2^* \\ B_3^* \\ B_4^* \\ B_5^* \\ B_6^* \\ B_7^* \\ B_8^* \\ B_9^* \\ B_{10}^* \\ B_{11}^* \\ B_{12}^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.02 \\ 72.465 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- Combining the cost and exergy balances, we obtain a general relationship that expresses the exergy cost of a flow as its exergy plus the sum of all the irreversibilities that have been generated in its production:

Direct Exergy Cost
Balance

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{a}_e \\ \mathbf{a}_x \end{pmatrix} \times (\mathbf{B}^* - \mathbf{B}) = \begin{pmatrix} \mathbf{I} + \mathbf{R} \\ 0 \\ 0 \end{pmatrix}$$

Exergy
Balance

$$\mathbf{B}^* = \mathbf{B} + \mathbf{A}^{(-1)} (\mathbf{I} + \mathbf{R})$$

The Cost–Irreversibility Formula

Notebook LM

$$B^* = B + \sum I$$

B^* = Exergy Cost
(Total physical burden).

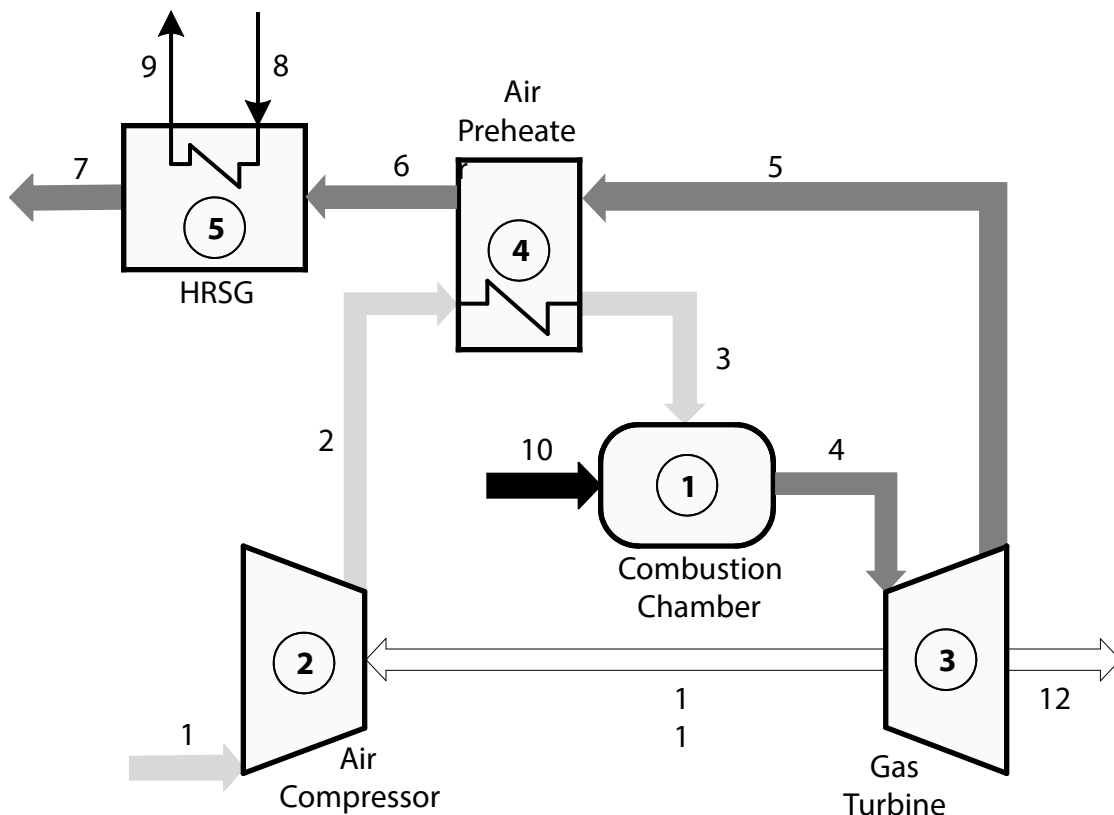
B = Remaining exergy content
of the product.

$\sum I$ = Cumulative irreversibilities
generated during production.

Takeaway: Every product carries an invisible physical burden—the total exergy permanently destroyed during its lifetime.

Exergy cost = useful exergy + accumulated irreversibility and allocated losses.

CGAM: Direct Exergy Costs

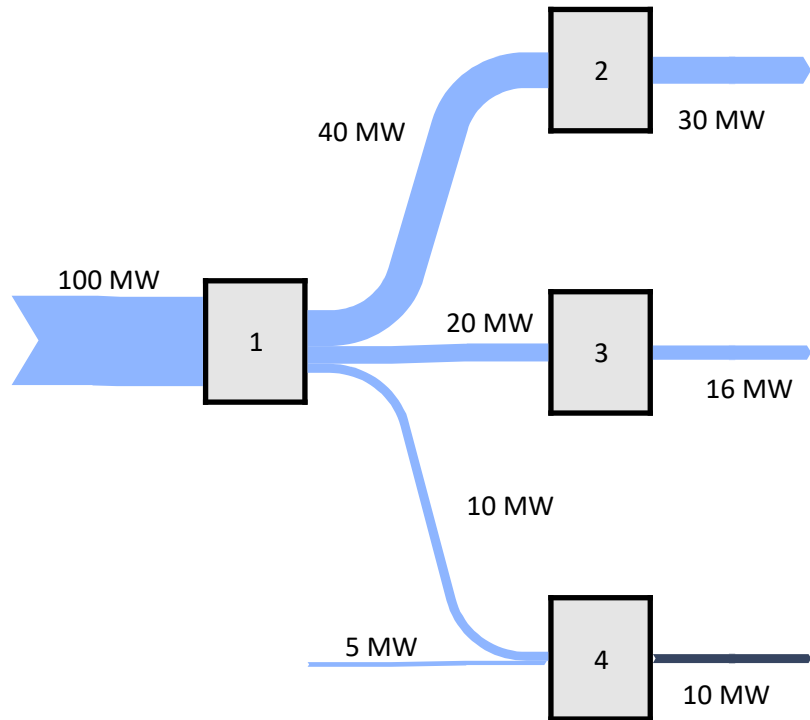


Flow	B (kW)	B* (kW)	k* (J/J)
1	0	0	1
2	28651	51022	1.781
3	50339	88815	1.764
4	102530	161280	1.573
5	38810	61048	1.573
6	14784	23255	1.573
7	2122	3339	1.573
8	19.43	19.43	1
9	9322	19936	2.139
10	72465	72465	1
11	31105	51022	1.64
12	30000	49210	1.64

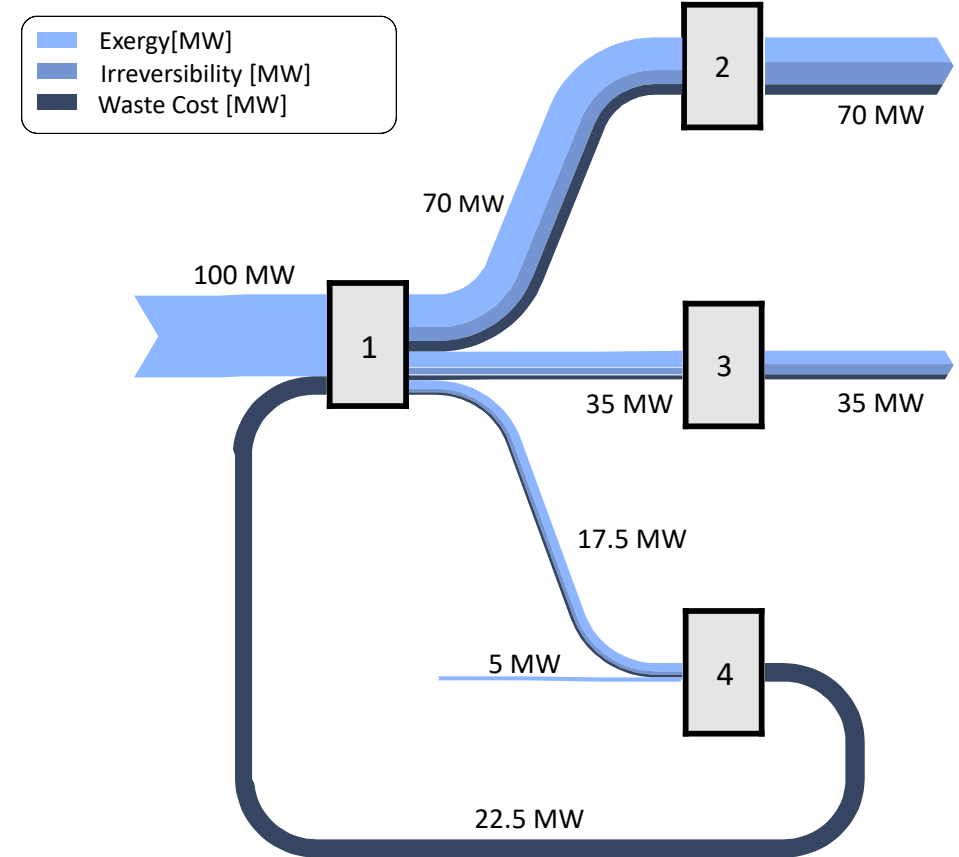
Requirements for Exergy Cost Calculation

- **The Physical Structure:** A system view as a set of processes connected by flows of mass and energy.
- **The Thermodynamic Model:** The thermodynamic properties, including exergy, of all flows in the system.
- **The Productive Structure:** The definition of efficiency of each individual process, as part of the system, and the waste cost allocation criteria.
- **The Economic Model:** The cost of external resources, equipment investment, and waste abatement costs.

From Exergy Flows to Cost Flows



**Exergy
Diagram**



**Exergy Cost
Diagram**

Monetary and Physical Input–Output Tables

- **Monetary Tables (MIOT):** These use economic values, such as billions of euros, to represent transactions. They form the basis for calculating Gross Value Added (GVA), which represents the wealth generated by each sector (wages, profits, taxes).
- They focus on aggregate economic activity. They are used to predict changes in the local or global economy in response to variations in final demand and to analyse the impact on the prices of final products when the costs of external resources change.
- **Physical Tables (PIOT):** These measure exchanges in units of matter or energy, such as kilograms (kg), cubic metres (m³) or megawatts (MW).
- They are essential for industrial ecology and environmental analysis. They enable the assessment of the direct and indirect consumption of natural resources and the environmental impact (emissions, waste, etc.) of a production system.

- The Fuel-Product Table is the connection between the Exergy Cost Theory and the Input-Output analysis of productive system.

$$\begin{pmatrix} \mathbf{A}_p \\ \mathbf{a}_e \\ \mathbf{a}_x \end{pmatrix} \times \mathbf{B} = \begin{pmatrix} \mathbf{P} \\ \mathbf{v}_e \\ 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{A}_p^{(-1)} & \mathbf{a}_e^{(-1)} & \mathbf{a}_x^{(-1)} \end{bmatrix} \times \begin{bmatrix} \mathbf{P} \\ \mathbf{v}_e \\ 0 \end{bmatrix} = \mathbf{A}_p^{(-1)} \mathbf{P} + \mathbf{a}_e^{(-1)} \mathbf{v}_e$$

$$\mathbf{A}_F \mathbf{B} = \mathbf{F} = \mathbf{A}_F \mathbf{A}_p^{(-1)} \mathbf{P} + \mathbf{A}_F \mathbf{a}_e^{(-1)} \mathbf{v}_e$$

$[FP]$	P_s	P
F_e		
F		

$$\mathbf{F} = \mathbf{F}_e + {}^t[\mathbf{FP}] \mathbf{u}$$

$$\mathbf{P} = \mathbf{P}_s + [\mathbf{FP}] \mathbf{u}$$

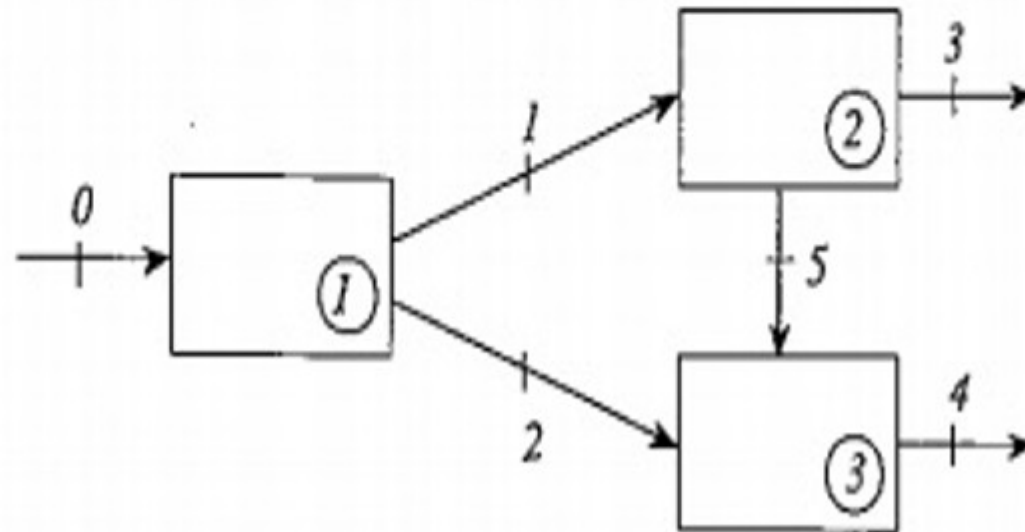
$${}^t[\mathbf{FP}] = \mathbf{A}_F \mathbf{A}_p^{(-1)} \hat{\mathbf{P}}$$

$${}^t\mathbf{F}_e = \mathbf{A}_F \mathbf{a}_e^{(-1)} \mathbf{v}_e$$

$${}^t\mathbf{P}_s = \mathbf{a}_p \mathbf{A}_p^{(-1)} \hat{\mathbf{P}}$$

A Three-Process Example

- ❖ Suppose a process like the one in the following figure:



Tracing Direct and Indirect Requirements

- ❖ We can define the **plant's incidence matrix** whose a_{ij} elements are "**1**" if the stream **j** enters subsystem **i**, "**-1**" if it exits, and "**0**" if they do not interact, then for the analyzed case:

$$A = \begin{bmatrix} +1 & -1 & -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & -1 & 0 & -1 \\ 0 & 0 & +1 & 0 & -1 & +1 \end{bmatrix}$$

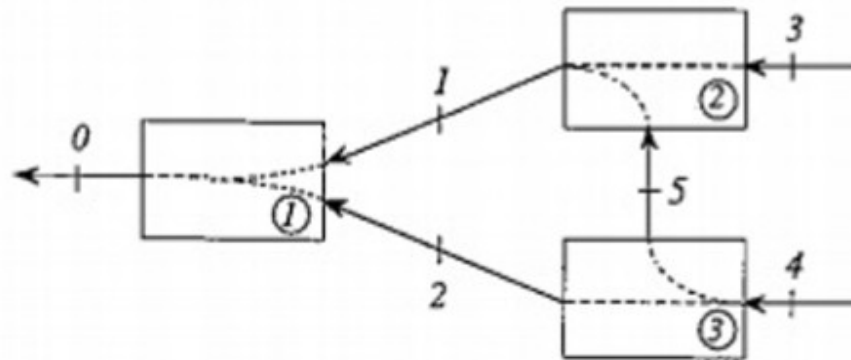
in general

$$A E^* = 0$$

- ❖ This is the **cost conservation equation**. Costs of the inputs of any component are transferred entirely among all the outputs.

The PRIMAL and DUAL of any process

- ❖ Every productive structure or linear interpretation of a system takes a **primal representation** to which **corresponds a single dual** and vice versa.
- ❖ While the arrows in the productive process point from the resources towards the products, the costs -or dual- point from the products towards the resources.



- ❖ **The cost seeks the origin, the products seek its destiny!**

Another Balance added to Thermodynamics

- ❖ Equilibrium Thermodynamics for open systems can simply be summarized in the following equations:

Mass Balance : $\mathbf{AM} = \mathbf{0}$

Enthalpy Balance : $\mathbf{AH} = \mathbf{0}$

Exergy Balance : $\mathbf{AEx} = \mathbf{I}$

Ex.Costs Balance : $\mathbf{AEx}^* = \mathbf{0}$

- ❖ where, $\mathbf{M}$, $\mathbf{H}$, $\mathbf{Ex}$ are vectors whose elements are mass, enthalpy and exergy of the material streams: m_i , $m_i h_i$ and $m_i(h_i - T_o s_i)$; $\mathbf{zero}$, $\mathbf{Q}$ and $\mathbf{Q}(1 - T_o/T)$ if they are heat flows; and $\mathbf{zero}$, $\mathbf{W}$, and $\mathbf{W}$, if they are working flows, respectively

■ Exergy

The Resources driven model

- ❖ The resources driven model allow to determine the system production and their costs as a function of:
 - The system resources: $\mathbf{F}_e \equiv (E_{01}, \dots, E_{0n})$
 - The efficiency of each process: $\mathbf{K}_D \equiv (F_i / P_i)_{i=1, \dots, n}$
 - The distribution ratios: $\langle \mathbf{FP} \rangle \equiv (E_{ji} / P_j)_{i,j=1, \dots, n}$
- ❖ The resources required in each process could be obtained as:

$$\mathbf{P} = (\mathbf{K}_D - \langle \mathbf{FP} \rangle)^{-1} \mathbf{F}_e$$

- ❖ The production cost could be obtained as:

$$\mathbf{C}_P = (\mathbf{U}_D - \langle \mathbf{FP} \rangle)^{-1} \mathbf{F}_e$$

The Demand driven model

- ❖ The demand driven model allow to determine the system resources cost as a function of:
 - The production demand: $\mathbf{P}_s \equiv (E_{10}, \dots, E_{n0})$
 - The efficiency of each process: $\mathbf{H}_D \equiv (P_i / F_i)_{i=1, \dots, n}$
 - The junction ratios: $\langle \mathbf{PF} \rangle \equiv (E_{ij} / F_j)_{i,j=1, \dots, n}$
- ❖ The resources required in each process could be obtained as:

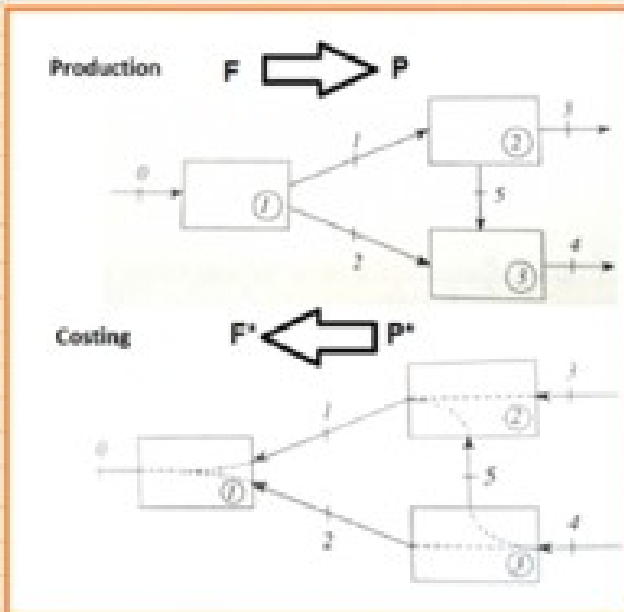
$$\mathbf{F} = (\mathbf{H}_D - \langle \mathbf{PF} \rangle)^{-1} \mathbf{P}_s$$

- ❖ The cost per production unit could be obtained as:

$$\mathbf{c}_P = {}^t (\mathbf{H}_D - \langle \mathbf{PF} \rangle)^{-1} \mathbf{c}_0$$

Notebook LM

Primal-Dual Mathematical Basis



The Primal System (Leontief)

- The physical exergy flows. Demand-driven.
- Evaluates the exergy required to satisfy production demands through the $\langle FP \rangle$ matrix.

The Dual System (Ghosh)

- The exact exergy costs. Supply-driven.
- Traces the propagation of resource costs through the $\langle PF \rangle$ matrix.

Key Insight: By replacing monetary flows with physical exergy flows, exergy costs propagate rigorously through complex supply chains without relying on arbitrary market prices.

TaesLab

TaesLab - Thermoeconomic Analysis of Energy Systems for MATLAB

TaesLab is a software tool designed for the thermoeconomic analysis of energy systems. It is distributed as a MATLAB/Octave Package.

It is based on the thermoeconomic analysis works developed since 1986 in the University of Zaragoza. Spain.



This tool is freely available through our Exergoecology Portal

Thank You

Physics, Economics and Responsible Resource Use